Conduction in Parallel

Abstract—*In this exercise, heat conduction in parallel is modeled. The solid is represented in two dimensions. The thickness of the two solids can be specified as an input. Coarse, medium, and fine meshes are available. The thermal conductivity of the two solids can be specified within limits. The temperature or heat flux at the left wall, and the temperature at the right wall can be specified. Temperature gradient and heat flux are reported. A plot of temperature along the interface is available. Contours of temperature can be displayed.*

1 Introduction

Thermal conduction is an important mode of heat transfer. Fourier's law of heat conduction relates the heat transfer rate to the temperature gradient, where thermal conductivity is represented by a proportionality constant. The temperature gradient adjusts so that the heat flux across the right wall is constant.

2 Modeling Details

The solid is represented in two dimensions by a rectangle. The procedure for solving the problem is:

1. Create the geometry.
2. Set the solid properties and boundary conditions.
3. Mesh the domain.

FlowLab creates the geometry and mesh, and exports the mesh to FLUENT. The boundary conditions and flow properties are set through parameterized case files. FLUENT continues to solve the problem until the convergence limit is satisfied or the specified number of iterations is achieved.

2.1 Geometry

The geometry is created from a set of four vertices. Edges are created over the vertices and are stitched with faces to facilitate meshing. The geometry consists of four walls and an interface (Figure 2.1).

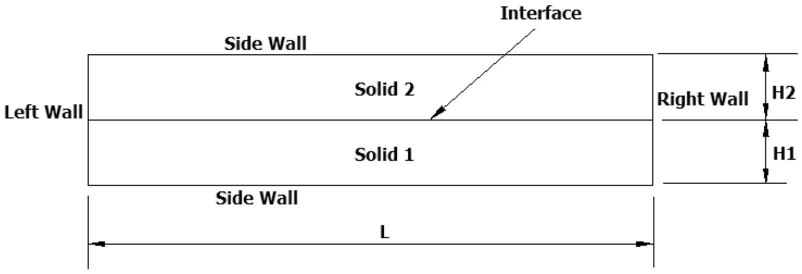

Figure 2.1: Schematic of the Solid with Boundaries

You have to specify the length of the two solids (L) and the thickness of two walls (H1 and H2).

2.2 Mesh

Coarse, medium, and fine mesh types are available. The discretization scheme used is based on the following logic:

- `$H` = Number of cells in the vertical direction for each solid
- `$L` = Number of cells in the horizontal direction

The numerical values for the two discretization parameters are given in Table 2.1.

Mesh Type	`$H`	`$L`
Coarse Mesh	2	10
Medium Mesh	3	20
Fine Mesh	4	30

Table 2.1: Mesh Discretization Logic

The face is meshed using the map scheme after the edges are discretized into intervals (Figure 2.2).

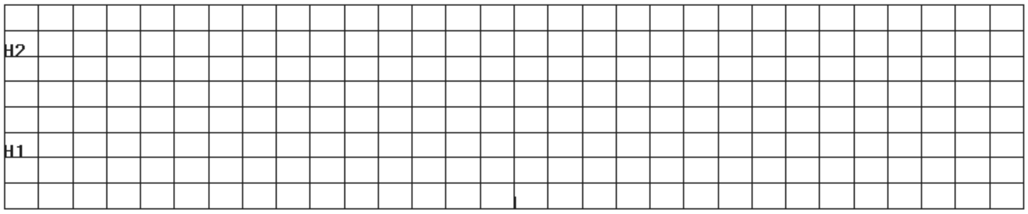

Figure 2.2: Mesh Generated by FlowLab

2.3 Physical Models for FLUENT

The energy equation is solved in FLUENT for the solid domain. Since there is no fluid flow, mass and momentum equations are not solved.

2.4 Material Properties

The default solid materials are Aluminum and Copper. The thermal conductivity of the solids can be specified within limits. Other materials such as Steel, Wood and a User Defined solid can also be selected.

2.5 Boundary Conditions

You can specify the following boundary conditions:

- Right wall temperature
- Left wall temperature *
- Left wall heat flux *

* *Based on the thermal condition (heat flux or temperature specification at the left wall).*

The following boundary conditions are assigned in FLUENT:

Left Wall	Wall
Right Wall	Wall
Interface between the solids	Interior
Side Walls	Symmetry

Table 2.2: Boundary Conditions Assigned to FLUENT

2.6 Solution

The mesh is exported to FLUENT along with the physical properties and the initial conditions specified. The material properties and the initial conditions are read through the case file. Instructions for the solver are provided through a `journal` file. Once the solution is converged or the number of specified iterations is met, FLUENT exports data to a `neutral` file and to `.xy plot` files. GAMBIT reads the neutral file for postprocessing activities.

3 Scope and Limitations

The maximum temperature allowed by FLUENT (hence FlowLab as well) is 5000 K. If the exercise temperature exceeds this limit, the temperature will be artificially restricted to 5000 K. Results obtained for cases where the temperature exceeds this limit may not be correct.

Difficulty in obtaining convergence or poor accuracy may result if input values are used outside the upper and lower limits suggested in the problem overview.

4 Exercise Results

4.1 Reports

The following reports are available:

- Heat flux
- Average temperature gradient

The average temperature gradient is calculated by taking the difference between the average temperature at left and right walls and dividing it by the total length of the domain.

4.2 XY Plots

The plots reported by FlowLab include:

- Residuals
- Temperature distribution

Figure 4.1 presents temperature across the interface between Solid 1 and Solid 2.

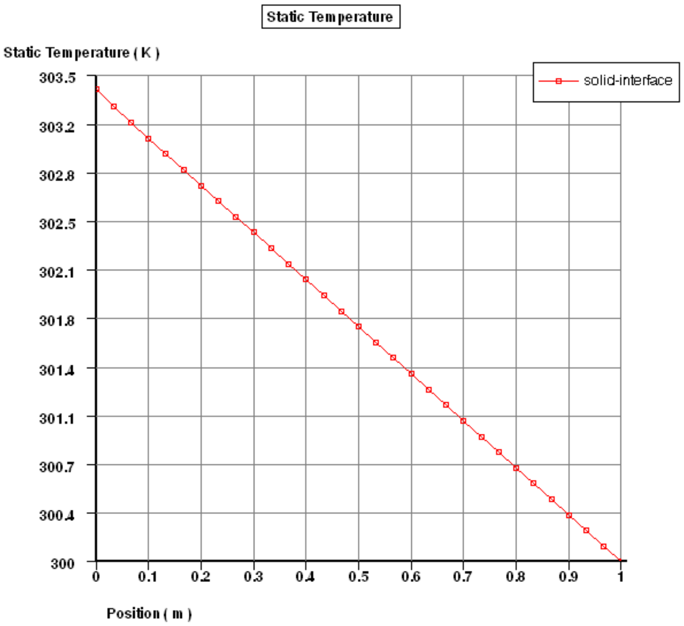

Figure 4.1: Temperature Distribution along Two Solids

4.3 Contour Plots

A contour plot of temperature is available (Figure 4.2).

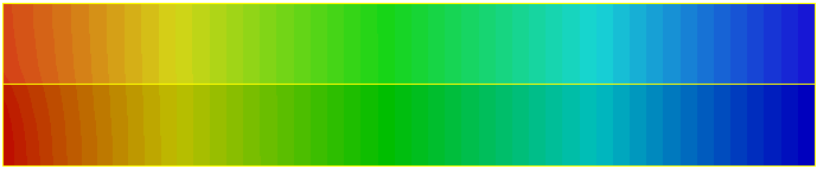

Figure 4.2: Contour Plot of Temperature

5 Verification of Results

The net thermal resistance for conduction in parallel is given by:

$$\frac{1}{\sum R} = \frac{1}{R_A} + \frac{1}{R_B} = \frac{k_A A_A}{L} + \frac{k_B A_B}{L} \tag{5-1}$$

The heat rate is given by:

$$q_x = \frac{\Delta T}{\sum R} = \frac{(T_1 - T_2)}{\sum R} \tag{5-2}$$

The net heat flux is determined by dividing the net heat rate by the total area.

The following default settings are used to verify the CFD model:

$L_1 = 1\ m$
$A_A = 0.1\ m^2$
$A_B = 0.1\ m^2$
$k_A = 202.4\ W/(m-K)$
$k_B = 387.6\ W/(m-K)$
$T_2 = 300\ K$

Table 5.1 compares the heat flux reported by FlowLab to theoretical values determined using the parameters summarized above. The FlowLab results were generated using the fine mesh option and a convergence criterion of `1e-12`.

Left Wall Temperature (K)	**Heat Flux (W/m^2)**	
	FlowLab	**Theory**
400	29500	29500
500	59000	59000
1000	206500	206500
4000	1091500	1091500

Table 5.1: Heat Flux Verification

6 Sample Problem

1. Run the cases using the default exercise settings and all three mesh options. It will be observed that the temperature gradient is slightly influenced by mesh density. Compare the results obtained using FlowLab with theoretical results.

2. Set the left wall boundary condition to the *wall temperature* type and run additional cases using the default material properties and all three mesh options. Compare the results obtained using FlowLab with theoretical results.

7 Reference

[1] *Incropera, F. P., and DeWitt, D. P.*, "Fundamentals of Heat and Mass Transfer", 4th Ed., Ch. 3.